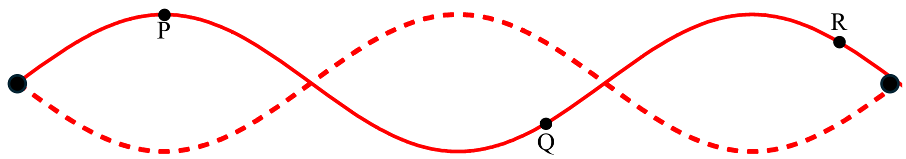
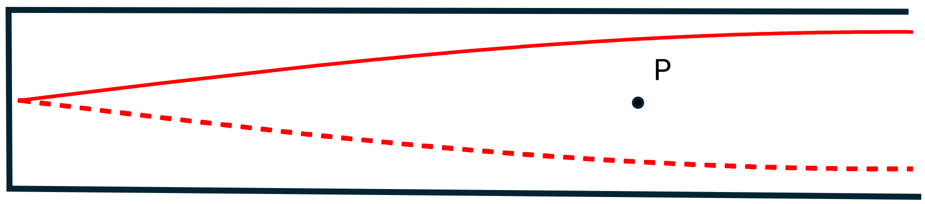
Standing Waves

The diagram shows a standing wave on a string with both ends fixed. The solid line shows the wave at time t = 0 when particle P in the string has maximum displacement and the dashed line shows the wave when P is at maximum negative displacement.



(a) Outline how a standing wave is formed.
(b) Describe the motion of particles P, Q and R in the string.
(c) State and explain which of particles P, Q and R has the largest average speed during one period.
(d) State the phase difference between P and Q and between P and R.
(e) Draw, approximately, the shape of the string at time $t = \frac{T}{8}$.
(f) The length of the string is 0.60 m and the speed of a travelling wave on this string is 120 m s^{-1}.
 (i) Calculate the frequency of this standing wave.
 (ii) State the frequency of the first harmonic on this string.
 (iii) State the time corresponding to the standing wave shown as the dashed line.
(g) The frequency of the standing wave is increased so that the fifth harmonic is established on the string.
 (i) State the frequency of the fifth harmonic.
 (ii) State how many nodes there are in the fifth harmonic.

The standing wave shown in the diagram is established in a pipe with one open and one closed end. The solid line is the wave at time t = 0 and the dashed line shows the wave half a period later.

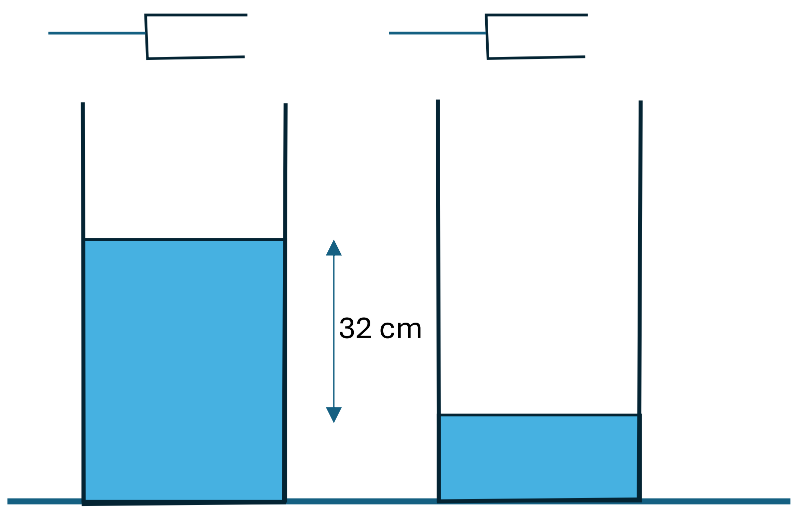


(h) Describe in as much detail as you can what this diagram represents exactly.

(i) P is the **equilibrium** position of a particle in the pipe. What is the approximate position of P at $t = 0$?

(j) Two consecutive harmonics in a pipe with one end closed and the other open are 300 Hz and 420 Hz. What is the frequency of the first harmonic in this pipe?

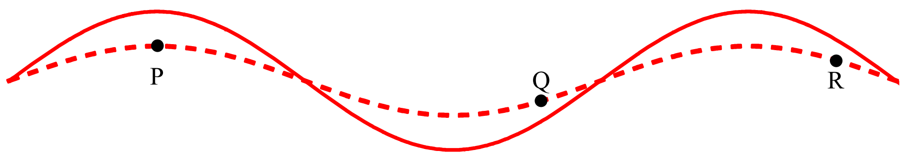
(k) A tuning fork of frequency 512 Hz sounds over a tube partially filled with water. Resonance is established in the tube. Water is slowly removed from the tube until resonance is again established. The water level is lowered by 32 cm.



Estimate the speed of sound.

Answers

(a) When two identical waves travelling in opposite directions superpose.
(b) They perform simple harmonic oscillations with the same period but different amplitude.
(c) P because it travels the largest distance in the same time.
(d) P and Q is π and P and R is zero.
(e)



(f)

(i) The wavelength is $\frac{2}{3}\times 0.60 = 0.40\,\text{m}$. Hence $f = \frac{120}{0.40} = 300\,\text{Hz}$ for the third harmonic.

(ii) The first harmonic then has frequency 100 Hz.

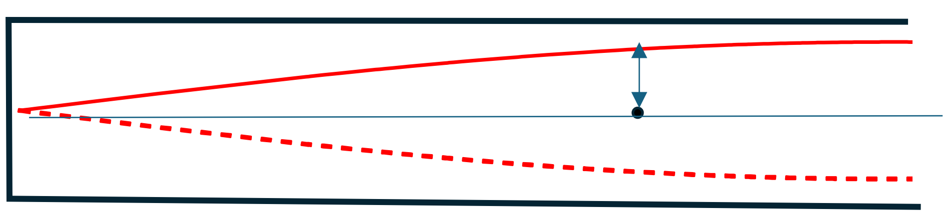
(iii) Half a period later i.e. 1.7 ms.

(g)

(i) 500 Hz.

(ii) Six.

(h) This is mathematical representation of the standing wave. The blue line shows the equilibrium situation when no wave is in the tube. The deviation from this line is proportional to the displacement of a particle in the pipe. But the displacement is horizontal because sound is longitudinal.



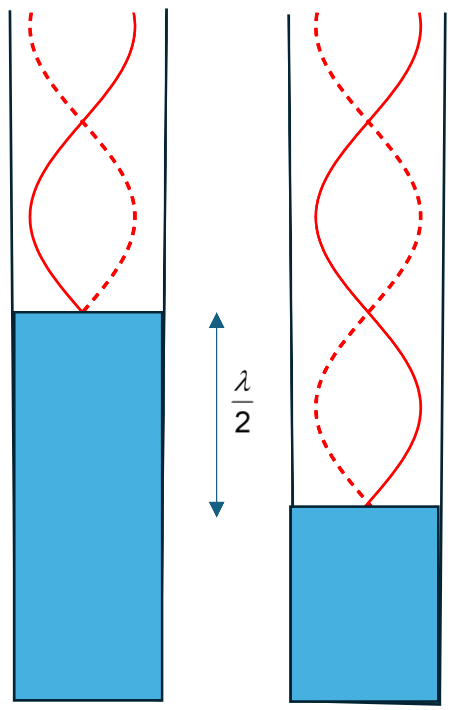
(i) On the blue line, some distance to the right of the equilibrium position.

(j) The two frequencies must be consecutive odd multiples of the first harmonic frequency so that is 60 Hz: 5×60 = 300 Hz and 7×60 = 420 Hz. ***OR***

$(2n-1)f = 300$
$(2n+1)f = 420$

Subtracting, $2f = 120$ so f = 60 Hz.

(k) The diagram shows that the water level must be reduced by half a wavelength. (It does not matter what we assume the wave to be in the first tube.)



So $\lambda = 0.64$ m and $c = \lambda f = 0.64 \times 512 = 330$ m s^{-1}.